

Exercise 161

For the following exercises, solve the trigonometric equations on the interval $0 \leq \theta < 2\pi$.

$$2 \cos \theta \sin \theta = \sin \theta$$

Solution

$$2 \cos \theta \sin \theta = \sin \theta$$

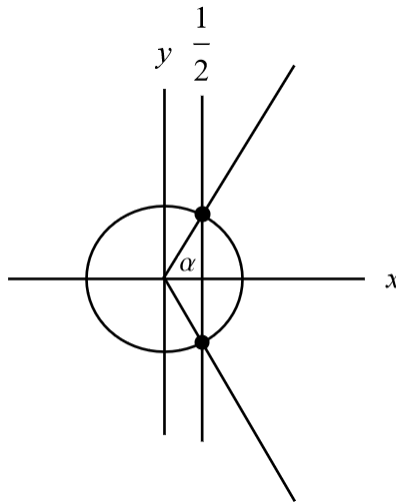
$$2 \cos \theta \sin \theta - \sin \theta = 0$$

$$\sin \theta(2 \cos \theta - 1) = 0$$

$$\sin \theta = 0 \quad \text{or} \quad 2 \cos \theta - 1 = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \cos \theta = \frac{1}{2}$$

0 and π satisfy $\sin \theta = 0$. The aim is to find the angles to the two points on the unit circle that are $1/2$ units to the right.



Taking the inverse cosine of $1/2$ gives 60° , or $\pi/3$ radians. This is α in the figure.

$$\alpha = \frac{\pi}{3}$$

The angle to the point below it is the same but negative, $-\pi/3$. Since every angle has to be between 0 and 2π , add 2π to it.

$$-\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}$$

Therefore,

$$\theta = \left\{ 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}.$$